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RESEARCH MEMORANDUM

AN ANALYSIS OF THE POTENTIALITIES OF A TWO-STAGE COUNTERROTATING SUPERSONIC COMPRESSOR

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

RESEARCH MEMORANDUM

AN ANALYSIS OF THE POTENTIALITIES OF A TWO-STAGE COUNTERROTATING SUPERSONIC COMPRESSOR

By Ward W. Wilcox

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SUMMARY

Because of recent developments which have indicated that relative Mach number limits may be raised to low supersonic values (<1.6) with satisfactory efficiency, an evaluation of the potentialities of an axial-flow counterrotating supersonic compressor was made. In order to determine the magnitude of the major design variables at the various pressure ratio levels, a one-dimensional analysis was made for blade speeds at intervals from 1000 to 1400 feet per second. The analysis showed that pressure ratios up to 8 may be obtained at values of turning angle, first-stage Mach number, and tip speed considerably lower than are now being considered for single-stage supersonic compressors. Although the second-stage Mach number is inherently higher than that of the first stage, it may be held to values of 1.6 or less for pressure ratios up to 6. For all cases considered by this analysis, the outlet conditions were such that the stator problem was minimized; that is, the Mach numbers were ≤ 1.2 and the flow angles were $\leq 30^{\circ}$.

The results of a simplified three-dimensional-design study are presented for a pressure ratio of 8 in order to demonstrate the similarities to and differences from the one-dimensional analysis. In general, the over-all trends are similar, although the average velocities were lower at the inlet and higher throughout the rest of the compressor than indicated by the one-dimensional analysis. For equal energy addition from hub to tip, more turning was required along the hub than along the tip in both stages; thus, the average turning was somewhat greater than that of the simplified analysis.

A comparison of the component performances of two compressor stages investigated at the NACA Lewis laboratory with the analytical requirements for a pressure ratio of 5 indicated that a counterrotating compressor with the following performance might be constructed from existing design information:

Weight flow, lb/(sec)(sq ft frontal area)	30
Pressure ratio.	5
Tip speed, ft/sec	1100
Adiabatic efficiency.	0.80 or higher

If the weight penalty resulting from the mechanical details of coaxial shafting were not too severe, a distinct advantage in engine size would be possible.

INTRODUCTION

The characteristics demanded of the compressor component by jet-engine designers are many and varied. The optimum compressor would pass maximum weight flow per unit frontal area, would produce the desired pressure ratio with a minimum number of stages and blades, would rotate at speeds suitable for the turbine component, and would maintain a high adiabatic efficiency over the full range of required operating conditions. Of the various types of compressor now being used or projected for future use, each has its merits and faults. The supersonic axial-flow compres- 2569
sor, for example, appears to offer advantages of high stage pressure ratio and high weight flows (reference 1) but problems peculiar to this compressor type have accompanied its introduction. For high single-stage pressure ratios (> 4) the problem of converting the high dynamic energy to useful pressure arises (reference 2). In addition, the high tip speeds and weight flows combine to create a difficult situation for the turbine designer (reference 3).

It has long been recognized that the counterrotation of successive stages of an axial-flow compressor offers potentially high pressure ratios per stage without excessive rotational speeds. At the tip speeds now utilized in jet engines, the mechanical problems of counterrotation limit the number of stages to two, driven by a single geared turbine or two free-running turbines through concentric shafts. In order to find any useful application, therefore, a counterrotating compressor must produce the desired pressure ratio in two stages and at the same time maintain the other desirable characteristics previously mentioned.

In the past, when aerodynamic considerations limited the Mach number relative to the blades to 0.8, the low over-all pressure ratios obtainable from two counterrotating stages did not warrant the additional mechanical complexity involved. Recent experimental results from compressors designed to operate at transonic relative Mach numbers (reference 4) have indicated, however, that an efficiency comparable with that of subsonic stages may be maintained up to a Mach number of 1.2. In addition, recent unpublished data show that Mach numbers in the low supersonic range (<1.6) may be employed in a supersonic axial-flow compressor with satisfactory adiabatic efficiency (0.85 for rotor alone). Because of these recent developments indicating that limiting Mach numbers can be raised well above previous values, a reevaluation of the possibilities of the counterrotating compressor appeared to be advisable.

With the belief that the trends determined by a one-dimensional analysis would also apply for practical compressor designs, such an analysis was made at the NACA Lewis laboratory for several typical counterrotating compressor designs in order to determine the pressure ratio potential and the order of magnitude of some of the required design variables at these pressure ratios. Complete velocity diagrams were determined at intervals of blade speed varying from 1000 to 1400 feet per second for three different outlet conditions. The first case specified purely axial discharge at a Mach number of 0.70 and thus eliminated the need for stators, although some annular diffusion would be necessary. The other two cases specified an outlet Mach number of 1.2 with absolute whirl components of 400 and 800 feet per second. The flow angles corresponding to these whirl components were approximately 15° and 30°, respectively. In addition, the effect of inlet guide vanes was determined for 10° counterprerotation at a blade speed of 1200 feet per second for the case with the outlet whirl component of 800 feet per second.

In order to indicate the validity of the trends established in the one-dimensional analysis, an approximate three-dimensional compressor design was made for a pressure ratio of 8. No attempt was made to determine the optimum configuration at this one pressure ratio. For this comparison, the blade speed of the one-dimensional analysis was considered as the mean outlet wheel speed of a three-dimensional design and the similarities and differences imposed by consideration of flow at other radii were determined.

In order to indicate the possibility of using current information to produce a practical counterrotating supersonic compressor, the component performance of two existing compressor rotors was applied to certain design specifications from the one-dimensional analysis for an over-all pressure ratio of 5.

ONE-DIMENSIONAL ANALYSIS AND CONDITIONS

Initial Assumptions

For the purposes of this analysis, vector diagrams were prescribed at the inlet and the outlet of each stage without regard to the specific details of blade construction necessary to obtain the desired conditions. The two stages were assumed to rotate in opposite directions at the same angular speed. Flow conditions were at constant radius throughout the analysis. Although some of the pressure ratios, and thus temperatures, covered by this analysis were high, a constant value of the specific heat of air (0.24) was used for simplicity.

In this report no direct consideration was given to the presence or absence of shock waves, boundary layer, flow separation, wakes, and so forth, but an allowance for the effect of such disturbances was made by assuming values of adiabatic efficiency based on experience. For the first stage, where Mach numbers are relatively low, a value of 0.90 was chosen. This value is considered to be realistic at low blade speeds

(<1200 ft/sec) but too high at higher blade speeds. Because the Mach number level in the second stage was considerably higher, an adiabatic efficiency of 0.85 was chosen. This efficiency was also considered realistic for Mach numbers up to 1.6, but too high for higher values at the present time. Although the assumption of a pressure recovery (ratio of outlet to inlet total pressure relative to the blades) as a function of relative Mach number may seem to be more suitable, this assumption results in increasing adiabatic efficiency with an increase in rotor turning (pressure ratio). This situation is contrary to compressor experience.

A typical set of vector diagrams for a counterrotating compressor without inlet guide vanes is shown in figure 1. Air enters the blading at a relative velocity V_1' and angle β_1' which depend on the blade speed ωr and the inlet velocity $V_{z,1}$. (All symbols are defined in appendix A.) For this analysis, the axial inlet Mach number was chosen to be 0.70 and blade speeds were varied from 1000 to 1400 feet per second. The air is decelerated relative to the blades and turned to the angle β_2' shown in the second diagram. Addition of the blade speed determines the absolute outlet velocity V_2 at the angle β_2. 2569

The amount of deceleration and turning in the blading of the first stage determines the pressure ratio obtainable from this stage and also establishes the whirl component at the entrance to the counterrotating second stage. In general, the deceleration in the first stage should be as large as possible to reduce the second-stage inlet Mach number. As a result of a survey of runs of several rotors, a value of deceleration was chosen which corresponded to 40-percent conversion to static pressure of the difference between relative static and total pressures at the rotor inlet. This is considered to be an attainable value of conversion at moderate inlet Mach numbers.

The wheel speed of the second stage, which rotates in a direction opposite to the first, then determines the value of angle and velocity relative to the second rotor. Outlet conditions for the second rotor were set up for the three different cases shown in figure 1 as selected on the basis of stator inlet conditions. From the established absolute outlet vectors it was possible to determine the relative conditions and hence the turning and velocities throughout the diagram.

The three different types of outlet conditions considered are:

Case 1: Discharge at an absolute Mach number of 0.70 in an axial direction. No stators are needed.

Case 2: Discharge at a Mach number of 1.2 with a whirl component of 400 feet per second. This condition results in a small absolute outlet angle β_4 of about 15°.

Case 3: Discharge at a Mach number of 1.2 with a whirl component of 800 feet per second. This condition results in an absolute outlet angle of about 30°.

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In cases 2 and 3, where stators are required to return the air to the axial direction, it was considered reasonable to assign a higher Mach number at the second-stage rotor outlet, since this change essentially transfers part of the deceleration to the stators. A sample calculation, which presents a complete example of case-2 design at 1200 feet per second, is given in appendix B. The pressure ratios presented in this report do not include stator losses.

Case 1

In the following discussion the major design variables which will be presented as functions of over-all pressure ratio for case 1 are: relative turning angle for each stage, Mach numbers relative to the inlet and the outlet of each stage, and the distribution of total-pressure ratio and enthalpy rise between stages. Similar presentations will be made for cases 2 and 3.

Required turning per stage. - The variation of both first- and second-stage relative turning angle with over-all pressure ratio is given by figure 2 for blade speeds of 1000, 1100, 1200, 1300, and 1400 feet per second. As would be expected, the over-all pressure ratio increases as first-stage turning and blade speed increase. Values of second-stage turning are restricted to a smaller range of values by the requirement of axial discharge at a Mach number of 0.7. The dash-dot line across the inlet turning curves represents axial discharge relative to the first rotor. In general, for case 1, the first-stage turning exceeds the second-stage turning except at very low pressure ratios.

Mach number relative to the first stage. - The inlet and the outlet relative Mach numbers are given for the first stage in figure 3. The inlet relative Mach number, which has a unique value for each blade speed, is quite low (<1.49) because blade speeds are kept low while the desired pressure ratios are still obtained. The outlet relative Mach number, which was determined by the requirement of 40-percent conversion mentioned in a preceding section, decreases slightly with increasing over-all pressure ratio and increases slightly as the blade speed is raised. In all cases, however, the outlet relative Mach number is subsonic and requires some type of shock within the rotor.

Mach number relative to the second stage. - The inlet and the outlet Mach numbers relative to the second stage are shown in figure 4. By the nature of counterrotation, the second-stage inlet Mach number is higher than the first-stage inlet Mach number. Because the second-stage inlet Mach number is virtually independent of blade speed for case 1, a single curve is given for all speeds. The outlet Mach numbers, as with the first stage, decrease with decreasing blade speed and increasing pressure ratio. The deceleration within the second stage, as shown by the difference between inlet and outlet Mach numbers, increases somewhat as blade speed decreases.

Distribution of enthalpy rise between stages. - The distribution of enthalpy rise or work between the two stages is a matter of some concern to turbine designers. Figure 5 shows that, for case-1 design, where rotation is introduced in the first stage and then removed by the second, the total enthalpy rise is evenly divided between the two stages. In a counterrotating jet engine with separate coaxial shafts, the second compressor stage would be driven by the first turbine stage. To assign only half the work load to the higher-temperature shorter-bladed first turbine stage might result in excessively difficult design problems for the second turbine stage. It is desirable, therefore, to delegate a greater share of the energy addition to the second-stage compressor rotor and hence to the first turbine rotor. In order to afford a direct comparison of enthalpy-rise distribution, the first-stage enthalpy rise is plotted against second-stage enthalpy rise for cases 1, 2, and 3 in figure 5. Contours of constant over-all pressure ratio are shown by the dashed lines.

Distribution of pressure ratio between stages. - For case-1 design, with the energy addition the same for each stage, the second-stage pressure ratio must necessarily be less than that of the first stage, because the inlet temperature is higher. In addition, for this particular analysis, second-stage efficiency was assumed to be 85 percent as compared with 90 percent for the first stage. The resulting distribution of pressure ratio is independent of blade speed and is shown in figure 6.

Case 2

For case 2, the outlet conditions chosen were for an absolute Mach number of 1.2 with a residual whirl component of 400 feet per second. An outlet angle of approximately 15° resulted.

Required turning per stage. - As shown by figure 7, the increase in pressure ratio resulting from an additional 400 feet per second of residual whirl makes less first-stage turning necessary for a given over-all pressure ratio than for case 1 (fig. 2). On the other hand, the second-stage turning is increased considerably and is now of the same order of magnitude as the first-stage turning. The absolute values of turning are conservative as compared with single-stage supersonic compressors of similar pressure ratio.

Mach number relative to the first and second stages. - It is obvious that adding a whirl component at the end of the second stage will not alter the entrance conditions into the first stage. On the basis of a chosen pressure ratio, however, the first-stage outlet Mach number will vary a small amount from that shown in figure 3 because less first-stage energy addition is necessary. This effect is of minor importance and therefore the first-stage outlet Mach number is not given for cases 2 and 3.

For constant pressure ratio, the second-stage inlet Mach number may
2569 be seen (fig. 8) to decrease slightly as blade speed is increased. A comparison of the curves for case 2 with those for case 1 (fig. 4) reveals considerably lower second-stage inlet Mach numbers for case 2 for all pressure ratios.

The relative outlet conditions for case 2 are not directly comparable with those of case 1. In an effort to redistribute the deceleration between the rotor and the stators, the outlet Mach number was increased from 0.70 to 1.2. The rotor deceleration, as shown by the difference between inlet and outlet relative Mach numbers, is thus lower than for case 1. In addition, the deceleration is slightly less for the higher blade speeds.

The stator problem for the conditions of case 2 is a minor one as compared with impulse-type supersonic compressors (reference 3). The flow angle is of the order of 15° and thus the required turning is low. Sample calculations have shown that the complete loss of the 400 feet per second whirl component would decrease the pressure ratio by only 6 percent for the outlet condition of case 2. Diffusion of the axial component of velocity is therefore the principal objective in the stators. This can probably be done efficiently, since a Mach number of 1.2 has a negligible normal-shock loss. The over-all pressure ratios given for cases 2 and 3 do not include any stator losses.

Distribution of enthalpy rise and total-pressure ratio between stages. - Reference to figure 5 shows that for case 2, second-stage enthalpy rise exceeds that of the first stage by a fixed amount which depends on the blade speed as well as on the residual whirl.

The variation of first- and second-stage pressure ratios with over-all pressure ratio is given in figure 9. At low over-all pressure ratios, the second-stage pressure ratio exceeds that of the first stage, while at values above approximately 7, the opposite is true.

Case 3

For case 3 an outlet Mach number of 1.2 was specified, with a residual whirl component of 800 feet per second. This condition results in a flow angle in the neighborhood of 30°.

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<u>Required turning per stage</u>. - The curves of first- and second-stage turning against over-all pressure ratio for case 3 are given in figure 10. For most over-all pressure ratios the second-stage turning exceeds the first-stage turning. As compared with case 2 (fig. 7), much less first-stage turning is required for a given over-all pressure ratio and as a result more second-stage turning is required. As in cases 1 and 2, at a given blade speed the second-stage turning is little affected by the pressure ratio required; that is, a higher pressure ratio is obtained by increasing the first-stage turning.

<u>Mach number relative to the second stage</u>. - The inlet and outlet relative Mach numbers are presented in figure 11 for case 3 for various blade speeds and over-all pressure ratios. For a given pressure ratio, the condition of lower first-stage turning than for case 2 results in lower second-stage inlet Mach numbers. As for case 2, this inlet Mach number decreases with increasing blade speed. Examination of the outlet relative Mach numbers shows that they are lower than for case 2 but are relatively unaffected by blade speed or over-all pressure ratio. In spite of the reduced inlet Mach number at the inlet of the second stage, the deceleration within the stage is greater for case 3 than for case 2, although still less than for case 1.

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The considerable increase in second-stage turning and deceleration rate required of case-3 design, accompanied by an easing of first-stage design conditions, penalizes the second-stage design in spite of the reduced Mach number level.

<u>Distribution of enthalpy rise and total-pressure ratio between stages</u>. - From figure 5, it may be noted that for case 3 the enthalpy rise is much greater for the second stage than for the first stage. This difference is accentuated at the higher blade speeds. As shown by figure 12, the second-stage pressure ratio exceeds the first-stage pressure ratio for case 3 for all except high over-all pressure ratios. This difference is greater at the higher blade speeds.

Comparison of Design Variables for Cases 1, 2, and 3

Although direct comparison of the three cases must be tempered by consideration of the stator requirements, some general observations can be made. For all three cases, the inlet conditions into the first stage are considered reasonable ($M_1' < 1.49$). For most pressure ratios, the turning angle is kept down to values below the angle of turning to the axial direction. The deceleration rate chosen is considered practical for this range of Mach number. Actually, a very wide choice of turning angles and deceleration rates is available.

In the second stage, the peak Mach number required for a given over-all pressure ratio may be reduced by increasing the second-stage pressure ratio. In this event stators are required to remove the whirl component and may be made to share part of the deceleration. Obviously, any number of variations of deceleration and second-stage turning are possible for a given pressure ratio with different stator inlet conditions as a result.

2569 For example, in a design with case-1 outlet diagram, at a pressure ratio of 6 and blade speed of 1300 feet per second, the second-stage turning angle is 15.5° and the outlet relative Mach number is 1.15. By a reduction in the relative Mach number to 1.02 in the blades, an outlet absolute Mach number of only 0.5 is obtained with a second-stage turning of 7.3°.

In general, case-3 operation simplifies the first-stage design at the expense of that of the second stage. For high pressure ratios (>8) outlet angles of the order of that for case 3 may be required, and the stators will have to assume a large share of the deceleration.

In the region between case-1 and case-2 design there is a type of operation where the residual whirl could be ignored with less loss than could be expected from stators. The cases presented in this report are not necessarily optimum but are typical examples set up to cover a range of conditions. Actually, a wide range of outlet conditions which could be utilized for specific applications is available.

Effect of Prerotation

A comparison of a case-3 design is presented in figure 13 for a blade speed of 1200 feet per second with the same design having 10° prerotation against the rotor direction. As compared with the effect of blade speed or first-stage turning, the effects of prerotation are small. At a given blade speed and pressure ratio, prerotation increases the Mach number entering the first stage and decreases the Mach number entering the second stage (fig. 13(a)). Both first- and second-stage turning required for a given pressure ratio are decreased a few degrees (fig. 13(b)). First-stage pressure ratio (and therefore first-stage enthalpy rise) is increased by prerotation with a corresponding decrease in second-stage values (fig. 13(c)). For case-3 operation, prerotation may be of some help in properly distributing the total deceleration between stages. For case 1, however, prerotation depreciates an enthalpy distribution which is already poor.

At the high axial inlet Mach numbers (0.70) characteristic of supersonic compessors, the use of prerotation is limited because of choking in the guide vanes. Unpublished experiments have shown that in an annulus of constant area, only 10° or 12° turning is possible without choking for an axial inlet Mach number of 0.70.

THREE-DIMENSIONAL COMPRESSOR DESIGN

The preceding analysis is based on the premise that a fluid element is always at the same radius in its path through the compressor. Such is not the case for practical compressor designs and some discussion of 2569 the resulting variation from the analysis is required. In order to provide maximum weight flow per unit of frontal area for the compressor, a low hub-tip radius ratio is necessary for the first stage. For example, in order to obtain a weight flow per unit frontal area of 33.9 pounds per second at an axial Mach number of 0.70, a hub-tip radius ratio of 0.50 is required. As the air is turned toward the axial direction from the initial direction, more area is available between blades and must be reduced by converging the shrouds. Greater energy addition is possible if the mean radius of rotation is increased and therefore the inner shroud radius only is usually increased. For compressors with low hub-tip radius ratios and high over-all pressure ratios, the rapid rate of change of curvature of the hub causes severe pressure gradients from hub to tip. In addition, the usual reversal of curvature near the outlet further complicates the pressure gradients. Because lengthening the flow path to reduce the magnitude of the curvature results in a long heavy compressor, a more suitable solution might be to curve the outer shroud inward to introduce opposite pressure gradients. In this case, the mean radius will be increased by a smaller amount, and greater turning than for a constant outer diameter will be required for a given pressure ratio. Because the turning in the counterrotating compressor is not excessive, a slight increase may offer a reasonable compromise.

In any rotating compressor, a problem which is further aggravated by low hub-tip radius ratios is the desirability of equal energy addition from hub to tip. Because the blade speed at the lower radii is less, the relative turning must be greater for equivalent energy addition. In a rotor with radial-element blading, a long tail section would be necessary to provide extra turning at the root. At the lower tip speeds made possible by counterrotation, departures from radial-element blades may be made which could shorten the length of a rotor.

In addition, the higher velocities at the hub of the first-stage rotor at the outlet (resulting from radial equilibrium and greater turning) are a form of negative prerotation for the second-stage rotor which allows equal energy addition in the second stage with less turning than would otherwise be required. Also, the blade height at the inlet of the second-stage rotor is less than that of the first-stage rotor and therefore the variation in inlet conditions from hub to tip is less. Design conditions which result in equal energy addition from hub to tip across the second stage usually result in discharge flow angles which are quite uniform from hub to tip, a prerequisite for good stator design.

The results of an approximate three-dimensional design of a counter-rotating compressor for a pressure ratio of 8 at a tip speed of 1300 feet per second is presented in table I. Adiabatic efficiencies of 0.90 and 0.85 were assumed for the two stages, no guide vanes were used, and 40-percent conversion was assumed for the first stage. Radial equilibrium was assumed to exist at the outlet of each compressor stage. Corresponding values of the design variables from cases 1, 2, and 3 at a blade speed comparable to the mean radius of this compressor at discharge are shown for comparison.

First stage. - In the three-dimensional design, the hub-tip radius ratio at the inlet was 0.5 and the tip radius ratio was reduced to 0.9 in the first stage. The change in curvature along the hub was not excessive for the depth of wheel shown by table I. The assumption of conversion to static pressure of 40 percent of the difference between total and static pressure in the first stage resulted in a tip relative Mach number at the blade exit of 0.82. The turning in the first stage was 44° at the tip and 54° at the hub for the stage pressure ratio of 2.5. These values of turning are higher than those obtained from the one-dimensional analysis for a comparable first-stage pressure ratio. The inlet relative Mach number, which varies from 1.40 to 0.93 from tip to hub, is considerably lower, on the average, than those obtained from the one-dimensional analysis.

Second stage. - The inlet Mach number of the second stage increases from tip to hub because of both radial equilibrium in the previous stage and additional first-stage turning at the hub for equal energy addition. As a result, the average value is somewhat higher than would be indicated by the one-dimensional analysis for the same stage pressure ratio. The absolute Mach number at discharge shows a similar trend, although the values are still at a very reasonable level for stator design (<1.3). The radial distributions of second-stage inlet angle, turning, and outlet angle are all quite uniform. It should be noted that, although the nominal tip speed of the compressor is 1300 feet per second, actually the mean tip speed of the first stage is 1235 feet per second and of the second stage, 1203 feet per second as a result of the changes in tip radius.

In general, the three-dimensional design results validate the trends shown in the one-dimensional analysis.

PRESENT AND FUTURE POSSIBILITIES OF COUNTERROTATION

From the component efficiencies of two compressor units which have been tested at the NACA Lewis laboratory, the present day potentialities of a counterrotating axial-flow compressor may be examined. For a pressure ratio of 5 at a blade speed of 1100 feet per second, about 20° of first-stage and 53° of second-stage turning are required (fig. 10). Unpublished data for the rotor of reference 4, operating at 1100 feet per second with about 20° turning at the tip and with over 40-percent conversion, indicate an adiabatic efficiency of 0.90. From figure 11, the second-stage inlet Mach number for this design would be 1.57. Unpublished runs of a supersonic compressor operating at design speed in Freon-12 (dichlorodifluoromethane, a commercial refrigerant) show that with inlet Mach number of 1.55 and turning of about 60°, the rotor adiabatic efficiency was 0.85. In addition, the relative Mach number at discharge was only 0.75, which would result in a lower stator inlet Mach number than those used in this analysis.

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From these data then, it seems permissible to predict that a counterrotating supersonic compressor which would deliver a pressure ratio of 5 at an efficiency of more than 80 percent with weight flows of 30 pounds per second per square foot frontal area at the reasonable tip speed of 1100 feet per second could be built from existing information.

This hypothetical compressor unit would then match existing compressors on the basis of weight flow and efficiency at a pressure ratio of 5 and at a tip speed currently practical. The number of blades and stages required is nearly the minimum possible. The inertia of each of the two rotating elements would be very similar, which should help starting and acceleration problems. In addition, the gyroscopic forces on each rotor resulting from the turning of the unit in flight would oppose one another and thus reduce the over-all gyroscopic effect on the aircraft.

Specifically, the counterrotating supersonic axial-flow compressor could compete with the subsonic axial-flow compressor in terms of weight flow, efficiency, and pressure ratio at comparable tip speeds with much less length and fewer blades. This supersonic axial-flow compressor would more than double the flow capacity of the centrifugal-type compressor at lower stress levels and with better efficiency, and would require lower tip speeds, turning angles, and Mach numbers than the single-stage supersonic compressor, which should result in better efficiency.

From the design values of the approximate compressor example for a pressure ratio of 8 (table I), it may be seen that the first-stage flow conditions for higher pressure levels also lie within the scope of existing experience. In the second stage, however, the inlet relative Mach number is higher than has been obtained with single-stage units.

The efficient diffusion of air from Mach numbers of this order of magnitude is the subject of intensive study at present, in connection with ram-jet inlets as well as with stators (reference 2). Advances in this field of research would lead directly to increased pressure ratio potential for the counterrotating supersonic compressor.

RESULTS AND CONCLUSIONS

2569 An evaluation of the potentialities of an axial-flow counterrotating supersonic compressor was made. In order to determine the magnitude of the major design variables at the various pressure ratio levels, a one-dimensional analysis was made for blade speeds at intervals from 1000 to 1400 feet per second. From the one-dimensional analysis the following results were obtained:

1. High over-all total-pressure ratios were obtained in the counterrotating compressor without exceeding present aerodynamic limits.

2. For over-all pressure ratios between 4 and 8, the values of first-stage relative Mach number, first- and second-stage turning angle, and blade speed represented a lowering of limits as compared with existing single-stage supersonic compressors.

3. Mach numbers relative to the second-stage inlet were inherently high, but appeared reasonable (1.6) for over-all pressure ratios below 6.

4. Discharge angles were relatively low and might possibly be held low enough to eliminate stators altogether.

5. For purely axial discharge the over-all enthalpy rise was evenly distributed between stages. At greater discharge angles the second-stage enthalpy rise exceeded that of the first stage, although the pressure ratio did not in all cases.

6. Inlet guide vanes which imposed prerotation against the direction of rotation were of no great advantage on the basis of increasing energy addition and might be harmful to the enthalpy-rise distribution in some cases.

An approximate three-dimensional design validated the general trends given by the one-dimensional analysis. For the same stage pressure ratios, however, the average velocities were lower at the inlet and higher throughout the rest of the compressor than those given by the one-dimensional analysis. For equal energy addition from hub to tip, more turning was required along the hub than along the tip in both stages.

From component efficiencies of compressor stages presently available, it seems possible to design a counterrotating compressor for a pressure ratio of 5 which could compete successfully with known compressor types. If the weight penalty resulting from the mechanical problems of coaxial shafts were not too severe, a distinct advantage over existing types would be possible.

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APPENDIX A

SYMBOLS

The following symbols are used in this report:

a sonic velocity

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M Mach number, V/a

P total pressure

p static pressure

T total temperature

V velocity

β angle of air with respect to axis of rotation

γ ratio of specific heats

η_{ad} adiabatic efficiency

ωr tip speed

Subscripts:

1 first-stage inlet

2 first-stage outlet

3 second-stage inlet

4 second-stage outlet

a stagnation

z axial component

θ tangential (whirl) component

Superscript:

' relative

APPENDIX B

SAMPLE CALCULATION

The following assumptions were made:

Blade speed (both stages), ft/sec	1200
Inlet axial Mach number	0.70
Prerotation (negative), deg	10
Adiabatic efficiency, first stage	0.90
Adiabatic efficiency, second stage	0.85
Conversion in first stage, percent of difference between relative total and static pressure converted to static pressure	40
Outlet whirl component, first stage, ft/sec	1000
Outlet whirl component, second stage, ft/sec	400
Outlet absolute Mach number	1.2

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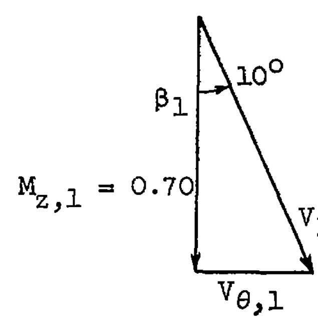
First Stage

With the inlet-guide-vane vector diagram shown, the absolute Mach



number is $0.70/\cos 10^{\circ} = 0.71$. At standard inlet conditions of pressure and temperature (from table I of reference 5), $a/a_a = 0.9531$ and sonic velocity $a_1 = 0.9531 \times 1117 = 1066$ ft/sec.

Thus,

$$V_{z,1} = 0.70 \times 1066 = 746 \text{ ft/sec}$$

$$V_1 = 0.71 \times 1066 = 758 \text{ ft/sec}$$

$$V_{\theta,1} = 758 \sin 10^{\circ} = 132 \text{ ft/sec}$$

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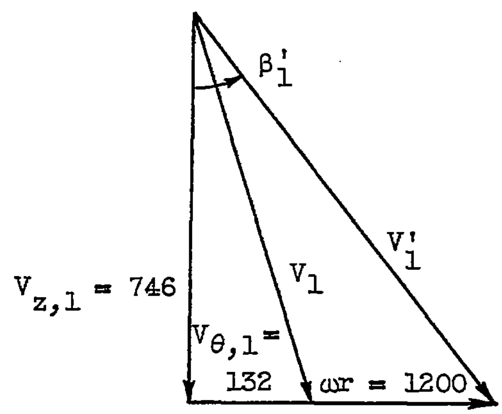
The first-stage inlet diagram at $\omega r = 1200$ then becomes



The angle of the relative velocity component

$$\beta_1' = \tan^{-1} 1332/746 = 60.7^{\circ}$$

and

$$V_1' = 746/\cos 60.7 = 1515 \text{ ft/sec}$$

$$M_1' = 1515/1066 = 1.429$$

For the criterion for the deceleration in the first stage, 40 percent of the difference between relative static and total pressure at the inlet was assumed to be converted to static pressure within the blades.

Thus,

$$p_2' = p_1' + 0.40\ (P_1' - p_1')$$

$$\frac{p_2'}{P_2'} = 0.60\ \frac{p_1'}{P_1'}\frac{P_1'}{P_2'} + 0.40\ \frac{P_1'}{P_2'} = \frac{P_1'}{P_2'}\left(0.60\ \frac{p_1'}{P_1'} + 0.40\right)$$

The ratio p_1'/P_1' may be found from tables for the Mach number M_1'. The ratio P_1'/P_2' is the inverse of the recovery factor for the blades. From the ratio p_2'/P_2', the relative Mach number at the blade discharge M_2' may be found.

In assigning an efficiency value to a supersonic compressor, a common method is to choose a value for the recovery factor of the blades P'_2/P'_1. However, as will be shown in appendix C, this assumption results in increasing values of adiabatic efficiency as the pressure ratio is increased. Of course, this condition is contrary to most experience in compressor work. It was concluded that a more realistic assumption would be that of constant adiabatic efficiency per stage.

The first-stage pressure ratio may be found as follows: 2563

$$\frac{P_2}{P_1} = \left[1 + \frac{\eta_{ad}\,(\gamma-1)\,\omega r\,(V_{\theta,1} + V_{\theta,2})}{a_{a,1}^{2}}\right]^{\frac{\gamma}{\gamma-1}}$$

$$= \left[1 + \frac{0.90\,(0.4)\,1200\,(132+1000)}{1117^{2}}\right]^{3.5}$$

$$= [1.3915]^{3.5} = 3.17$$

From this pressure ratio and the adiabatic efficiency of 0.90, the recovery factor may be found to be 0.899 from figure 14.

$$\frac{p'_2}{P'_2} = \frac{0.60 \times 0.3016 + 0.40}{0.899} = 0.646$$

$$\left.\begin{aligned} M'_2 &= 0.815 \\ a/a_a &= 0.9396 \text{ when } M'_2 = 0.815 \\ a/a_a &= 0.8425 \text{ when } M'_1 = 1.429 \end{aligned}\right\} \text{ from tables}$$

Since the stagnation sonic velocity relative to the blades remains unchanged,

$$a_2 = \frac{a_1 \times 0.9396}{0.8425} = 1190 \text{ ft/sec}$$

$$V'_2 = 1190 \times 0.815 = 970 \text{ ft/sec}$$

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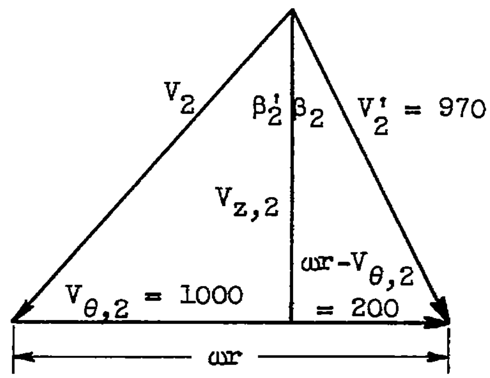
With the assumed value of $V_{\theta,2}$ of 1000 ft/sec, the outlet diagram becomes



The relative angle

$$\beta_2' = \sin^{-1} 200/970 = 11.9$$

The first-stage turning

$$\beta_1' - \beta_2' = 60.7 - 11.9 = 48.8^{\circ}$$

and

$$V_{z,2} = 970 \cos 11.9 = 951 \text{ ft/sec}$$

$$\beta_2 = \tan^{-1} 1000/951 = 46.5^{\circ}$$

$$V_2 = 951/\cos 46.5^{\circ} = 1380 \text{ ft/sec}$$

$$M_2 = 1380/1190 = 1.16$$

The total-temperature ratio

$$\frac{T_2}{T_1} = \left[1 + \frac{0.4 \times 1200\ (1132)}{1117^2}\right] = 1.436$$

Thus,

$$T_2 = 518.6 \times 1.436 = 744$$

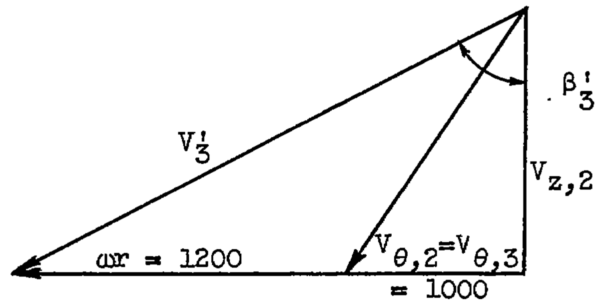
and

$$a_{a,2} = 49.01\sqrt{744} = 1338$$

[redacted]

Second Stage

Addition of the second-stage blade speed in the opposite direction results in the following diagram for the second-stage inlet:



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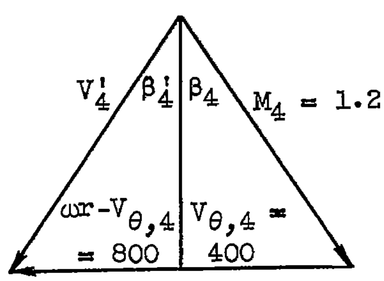
$$\beta_3' = \tan^{-1} 2200/951 = 66.6^{\circ}$$

$$V_3' = 951/\cos 66.6 = 2400$$

$$M_3' = 2400/1190 = 2.01$$

For the assumed outlet whirl component of 400 ft/sec the outlet diagram is



The second-stage temperature ratio is

$$\frac{T_4}{T_3} = 1 + \frac{(\gamma-1)\ \omega r\ (V_{\theta,3} + V_{\theta,4})}{a_{a,2}{}^2}$$

$$= 1 + \frac{(\gamma-1)\ 1200\ (1000 + 400)}{1336^2}$$

$$= 1.376$$

$$T_4 = 1.376\sqrt{744} = 1025^{\circ}\ R$$

$$a_{a,4} = 49.01\sqrt{1025} = 1570\ \text{ft/sec}$$

At a Mach number of 1.2,

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$$a/a_a = 0.8811\ \text{(tables)}$$

$$a_4 = 0.8811 \times 1570 = 1384\ \text{ft/sec}$$

$$V_4 = 1.2 \times 1384 = 1660\ \text{ft/sec}$$

$$\beta_4 = \sin^{-1} 400/1660 = 13.9^{\circ}$$

$$V_{z,4} = 1660\ \cos\ 13.9^{\circ} = 1615\ \text{ft/sec}$$

$$M_{z,4} = 1615/1384 = 1.168$$

Second-stage turning

$$\Delta\beta' = 66.6 - 26.3 = 40.3^{\circ}$$

$$V_4' = 1615/\cos\ 26.3 = 1801$$

$$M_4' = 1801/1384 = 1.3$$

Second-stage pressure ratio

$$\frac{P_4}{P_2} = \left[1 + \frac{(\gamma-1)\eta_{ad}\ \omega r\ (V_{\theta,3} + V_{\theta,4})}{a_{a,2}{}^2}\right]^{\frac{\gamma}{\gamma-1}}$$

$$= \left[1 + \frac{(\gamma-1)\ 0.85 \times 1200\ (1000+400)}{1336^2}\right]^{3.5}$$

$$= [1.32]^{3.5} = 2.642$$

Over-all pressure ratio

$$\frac{P_2}{P_1} \times \frac{P_4}{P_2} = 3.17 \times 2.642 = 8.38$$

APPENDIX C

DEVELOPMENT OF RELATION BETWEEN RECOVERY FACTOR AND ADIABATIC EFFICIENCY

In the design of supersonic compressors, the greatest single problem is the efficient diffusion of air relative to either rotor or stator blades. 2569 A large amount of the information pertaining to supersonic diffusion has been obtained in wind tunnels with stationary diffusers or blade cascades. It is customary to express the performance of such diffusers in terms of recovery factor, that is, the ratio of outlet total pressure to inlet total pressure, relative to the diffuser. In compressor work, recovery factor may be expressed in terms of known relations as follows:

$$\frac{P_2'}{P_1'} = \frac{P_2}{P_1} \times \frac{P_1}{P_1'} \times \frac{P_2'}{P_2}$$

The terms P_1/P_1' and P_2'/P_2 are point functions and may be transformed by isentropic relations as follows:

$$\frac{P_2'}{P_1'} = \frac{P_2}{P_1} \times \left(\frac{T_1}{T_1'} \times \frac{T_2'}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$$

Because the total temperature relative to the blades remains constant, $T_1' = T_2'$ and

$$\frac{P_2'}{P_1} = \frac{P_2}{P_1}\left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$$

From the definition of adiabatic efficiency,

$$\eta_{ad} = \frac{T_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{T_2 - T_1}$$

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$$\frac{T_2}{T_1} = \frac{\eta_{ad} + \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\eta_{ad}}$$

and by substitution,

$$\frac{P_2'}{P_1'} = \frac{P_2}{P_1}\left[\frac{\eta_{ad}}{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1 + \eta_{ad}}\right]^{\frac{\gamma}{\gamma-1}}$$

In figure 14 the relation between pressure recovery and adiabatic efficiency is given for a range of pressure ratios at intervals of adiabatic efficiency from 60 to 100 percent. The dashed lines of figure 14 represent contours of constant enthalpy rise and correspond to the pressure-ratio scale only when standard inlet conditions are assumed.

From figure 14 it may be seen that at low pressure ratios, small decreases in pressure recovery will result in larger decreases in adiabatic efficiency. As stage pressure ratio increases, adiabatic efficiency increases for constant values of pressure recovery.

REFERENCES

1. Wright, Linwood C., and Klapproth, John F.: Performance of Supersonic Axial-Flow Compressors Based on One-Dimensional Analysis. NACA RM E8L10, 1949.

2. Ferri, Antonio: Preliminary Analysis of Axial-Flow Compressors Having Supersonic Velocity at the Entrance of the Stator. NACA RM L9G06, 1949.

3. Stewart, Warner L., Schum, Harold J., and Whitney, Warren J.: Investigation of Turbines for Driving Supersonic Compressors. I - Design and Performance of First Configuration. NACA RM E52C25, 1952.

4. Lieblein, Seymour, Lewis, George W., Jr., and Sandercock, Donald M.: Experimental Investigation of an Axial-Flow Compressor Inlet Stage Operating at Transonic Relative Inlet Mach Numbers. I - Over-All Performance of Stage with Transonic Rotor and Subsonic Stators Up to Rotor Relative Inlet Mach Number of 1.1. NACA RM E52A24, 1952.

5. The Staff of the Ames 1- by 3-foot Supersonic Wind-Tunnel Section: Notes and Tables for Use in the Analysis of Supersonic Flow. NACA TN 1428, 1947.

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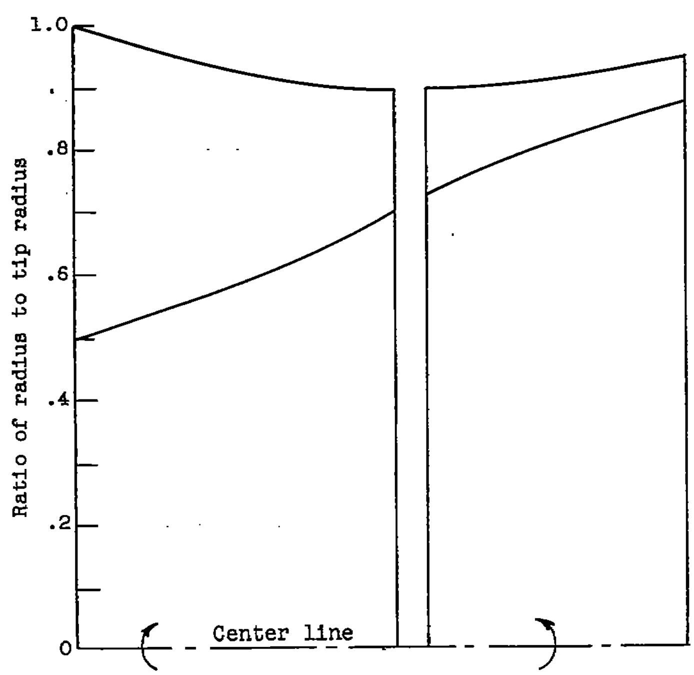
TABLE I - COMPARISON OF AN APPROXIMATE THREE-DIMENSIONAL DESIGN FOR A TIP SPEED OF 1300 FEET PER SECOND WITH ONE-DIMENSIONAL ANALYSIS AT A BLADE SPEED COMPARABLE WITH THAT AT THE OUTLET MEAN RADIUS

Concept	Three-dimensional design			One-dimensional design		
	Radius			Case		
	Tip	Mean	Hub	1	2	3
First stage						
Inlet radius ratio	1.0	0.791	0.5	1.0	1.0	1.0
Inlet angle, β_1', deg	60.2	54	41	58.2	58.2	58.2
Inlet Mach number, M_1'	1.404	1.189	.926	1.32	1.32	1.32
Turning angle, deg	43.7	50.3	53.8	60	48	35
Outlet angle, β_2, deg	44.5	43.3	48.6	54	48.5	44
Outlet radius ratio	.90	.811	.705	-----	-----	-----
Stage pressure ratio	2.525	2.445	2.54	3.45	2.93	2.44
Second stage						
Inlet angle, β_3', deg	66.2	61.5	62.2	70	68.5	68
Inlet Mach number, M_3'	1.95	2.06	2.15	2.21	2.05	1.89
Turning angle, deg	48	48.2	48.7	21	41.5	52
Outlet angle, β_4, deg	26.9	28	26.3	0	15	30
Outlet radius ratio	.95	.916	.884	-----	-----	-----
Outlet Mach number, M_4	1.20	1.29	1.29	.70	1.2	1.2
Stage pressure ratio	3.27	3.33	3.21	2.33	2.76	3.30
Both stages						
Over-all pressure ratio	8.26	8.15	8.16	8	8	8



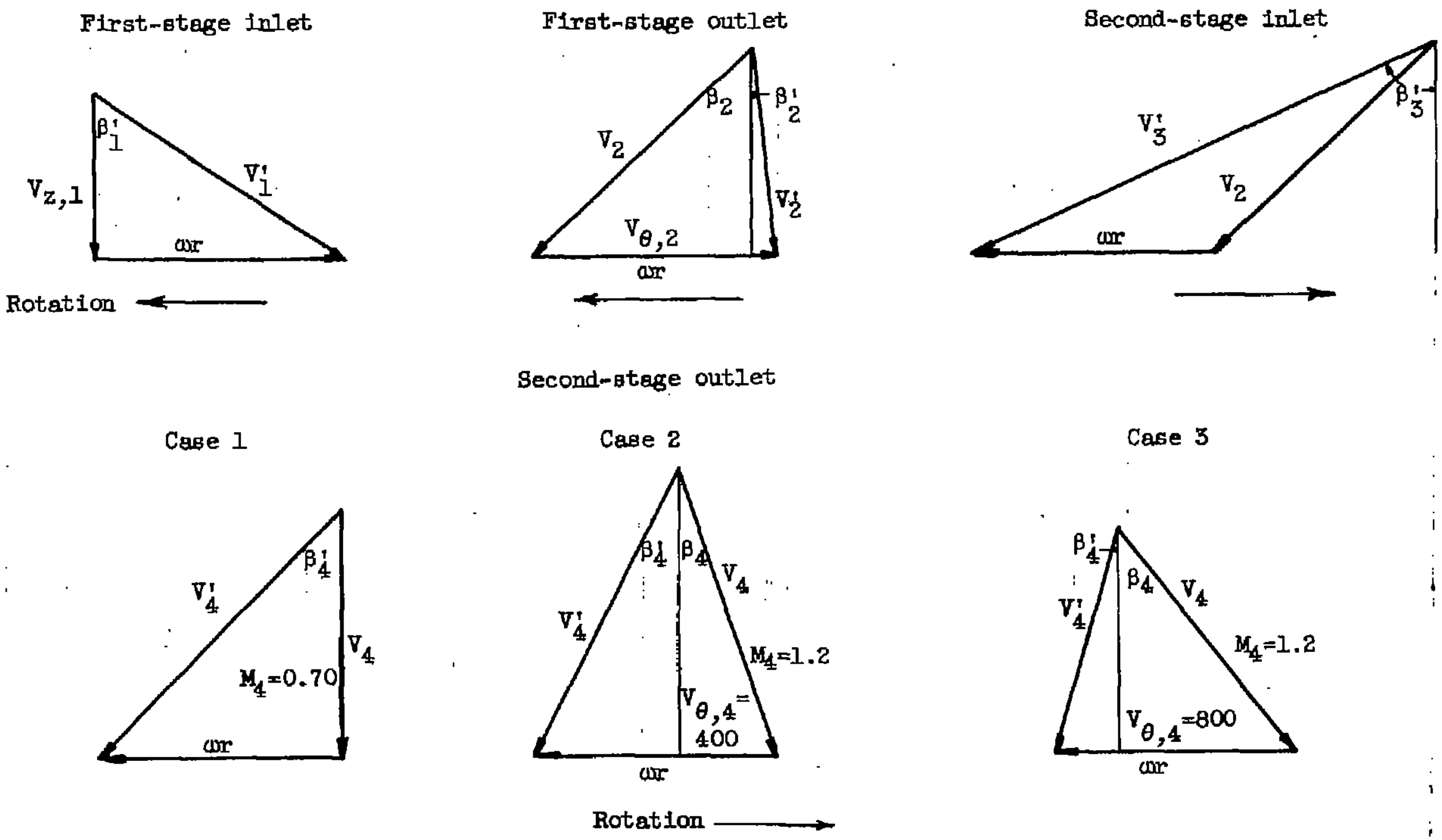


Figure 1. - Typical vector diagrams for counterrotating supersonic axial-flow compressors.

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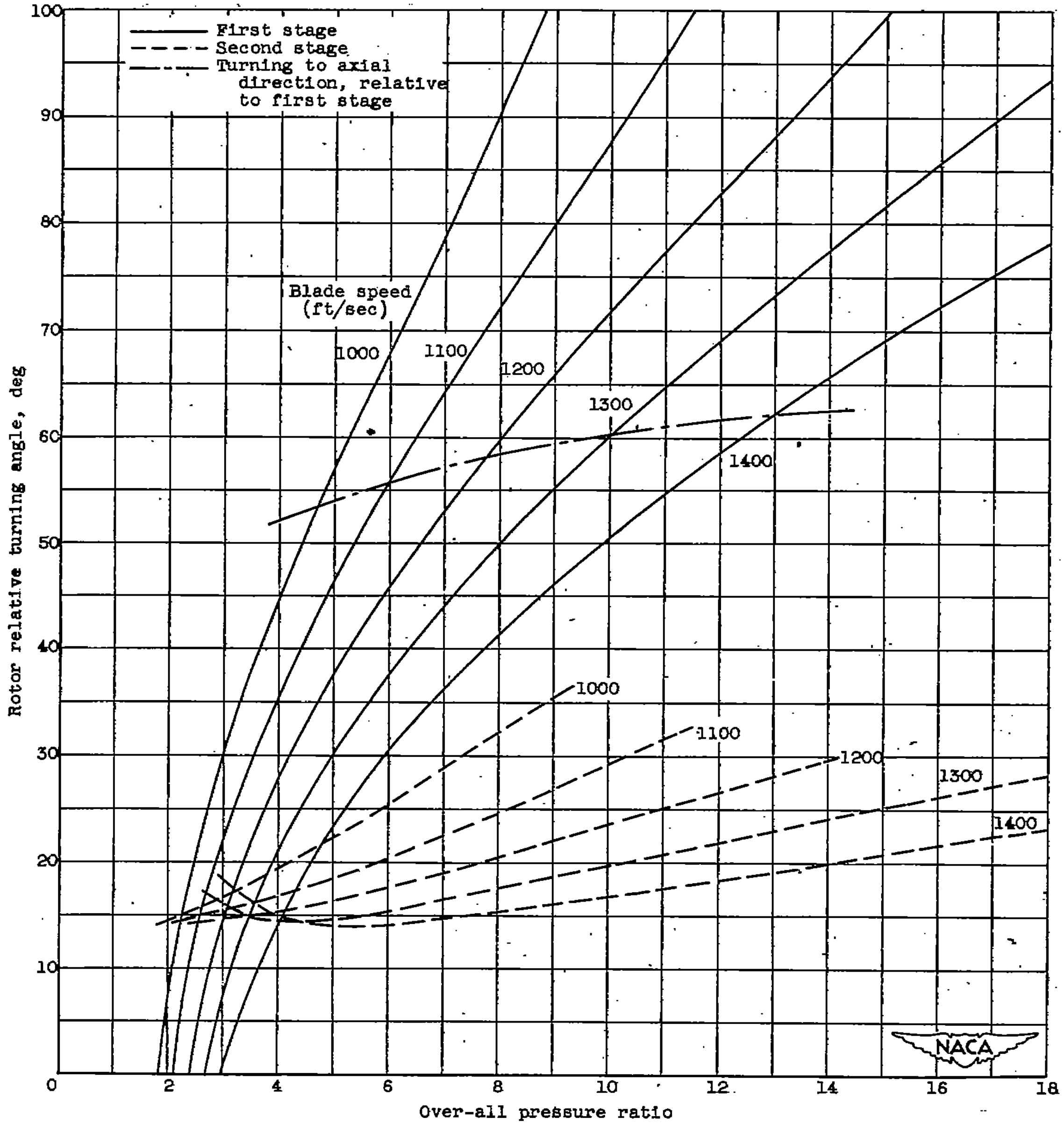

Figure 2. - Variation of rotor relative turning angle with over-all pressure ratio. Case 1.

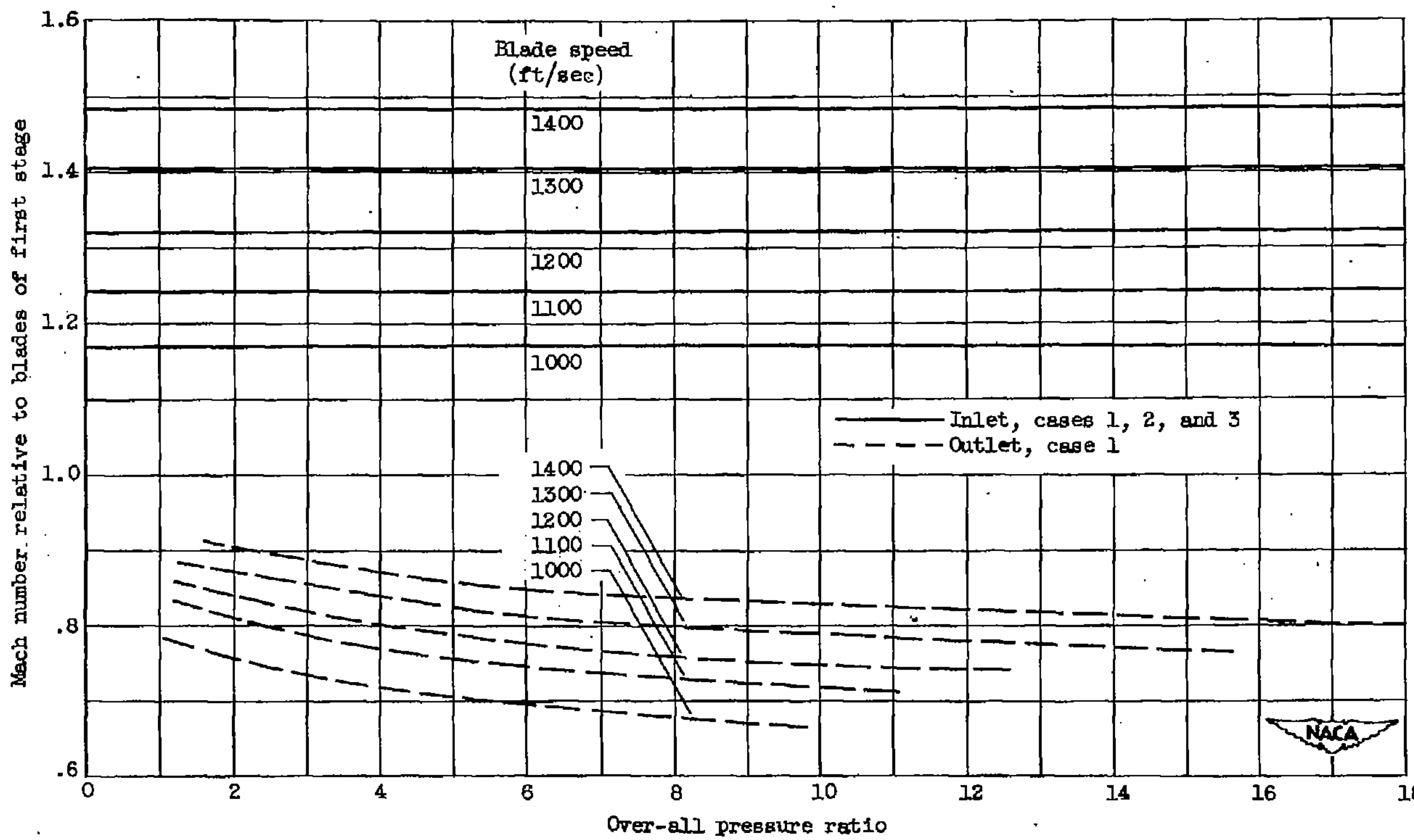

Figure 3. - Variation of Mach number relative to blades at inlet and outlet of first stage with over-all pressure ratio.

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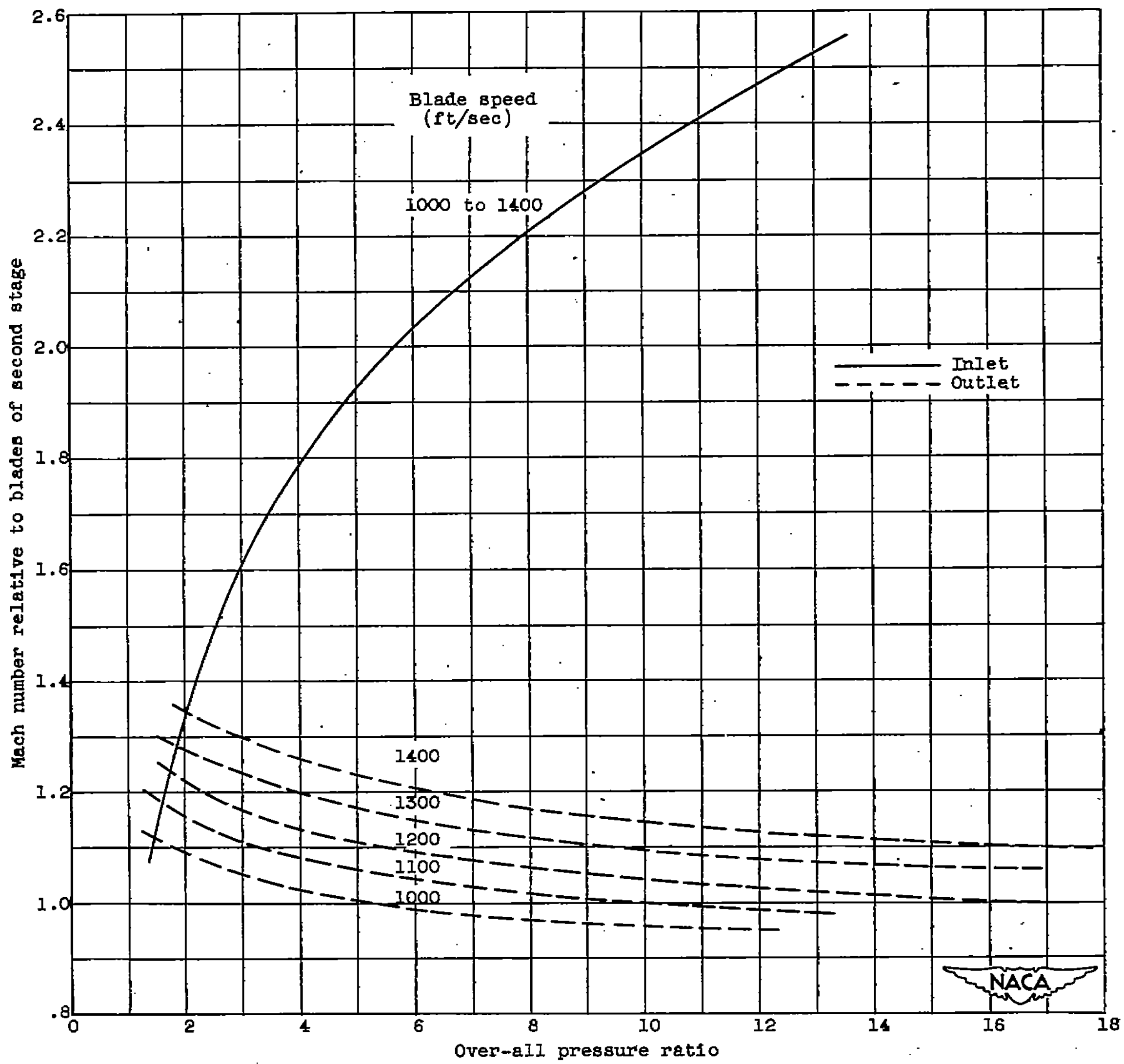

Figure 4. - Variation of inlet and outlet Mach numbers relative to blades of second stage with over-all pressure ratio. Case 1.

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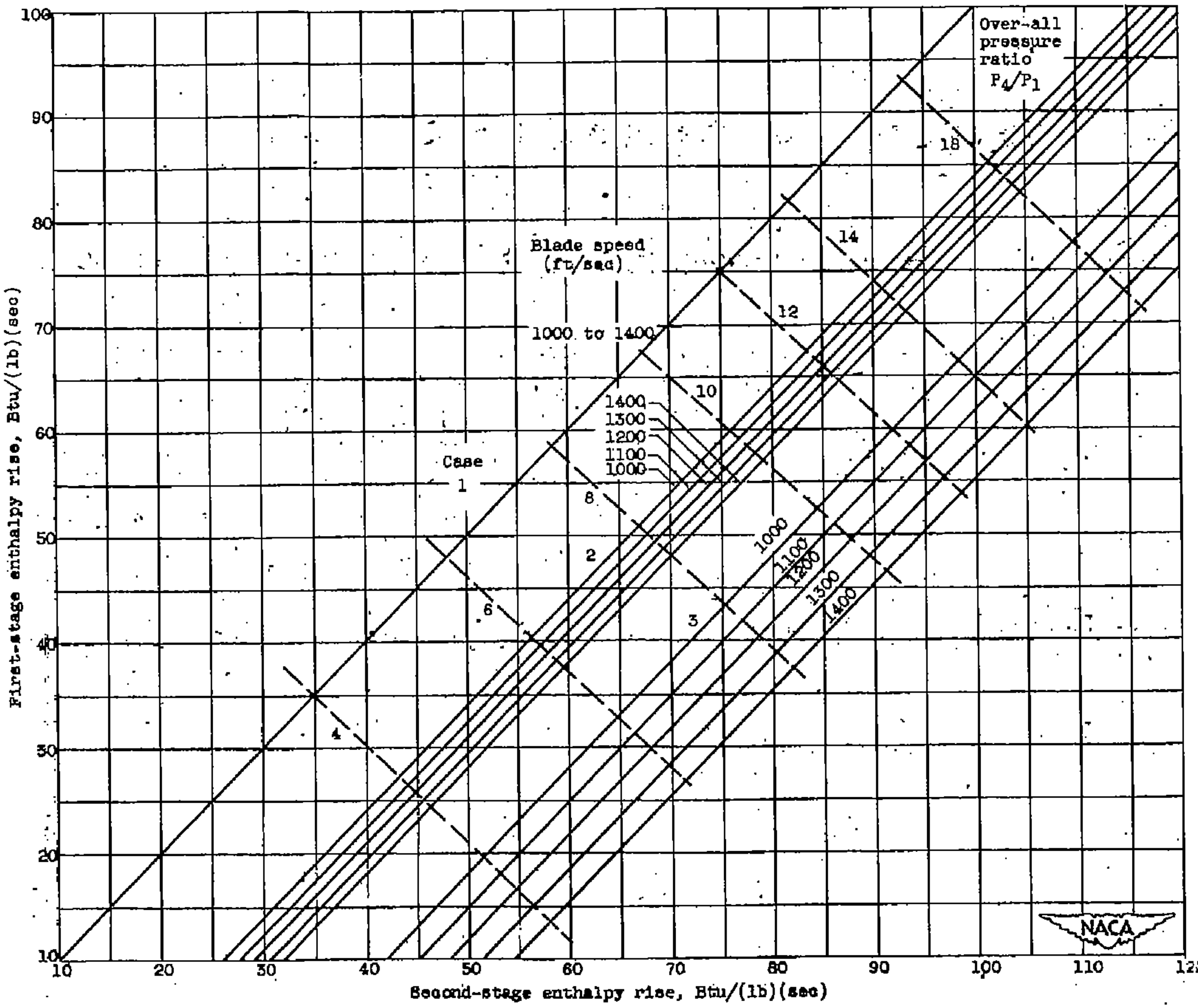

Figure 5. - Distribution of enthalpy rise between stages. Cases 1, 2, and 3.

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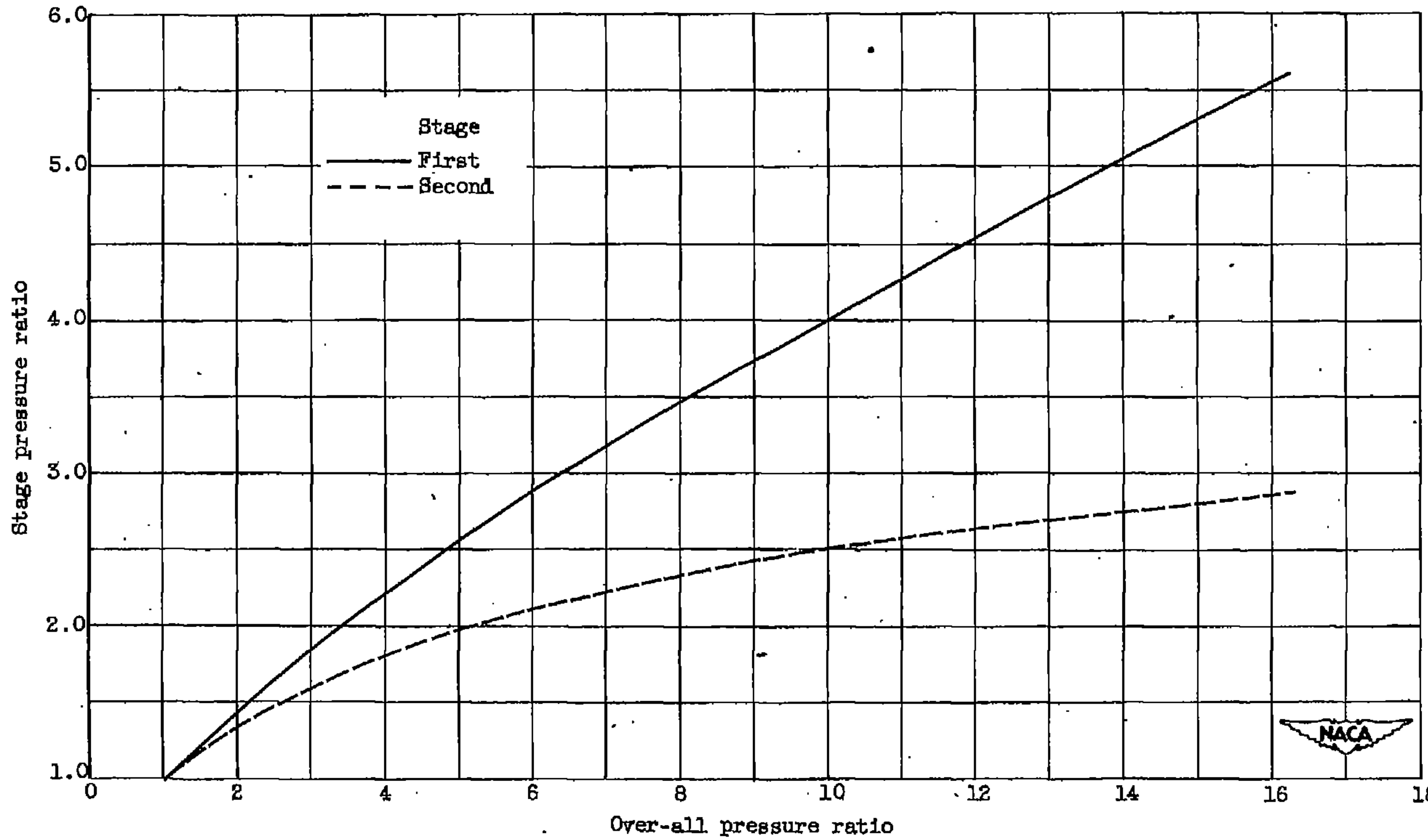

Figure 6. - Distribution of pressure ratio between stages. Case 1; blade speed, 1000 to 1400 feet per second.

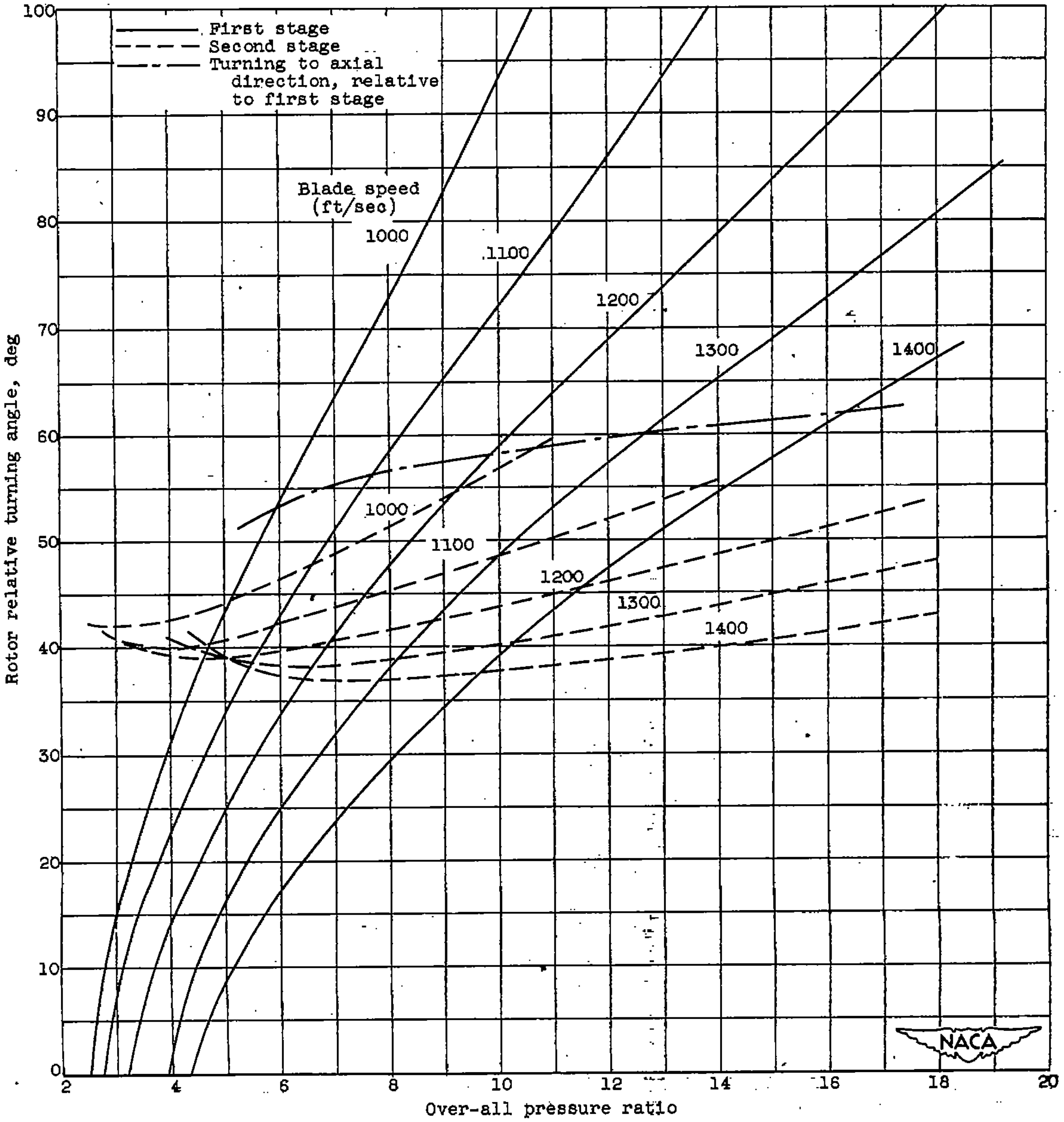

Figure 7. - Variation of rotor relative turning angle with over-all pressure ratio. Case 2.

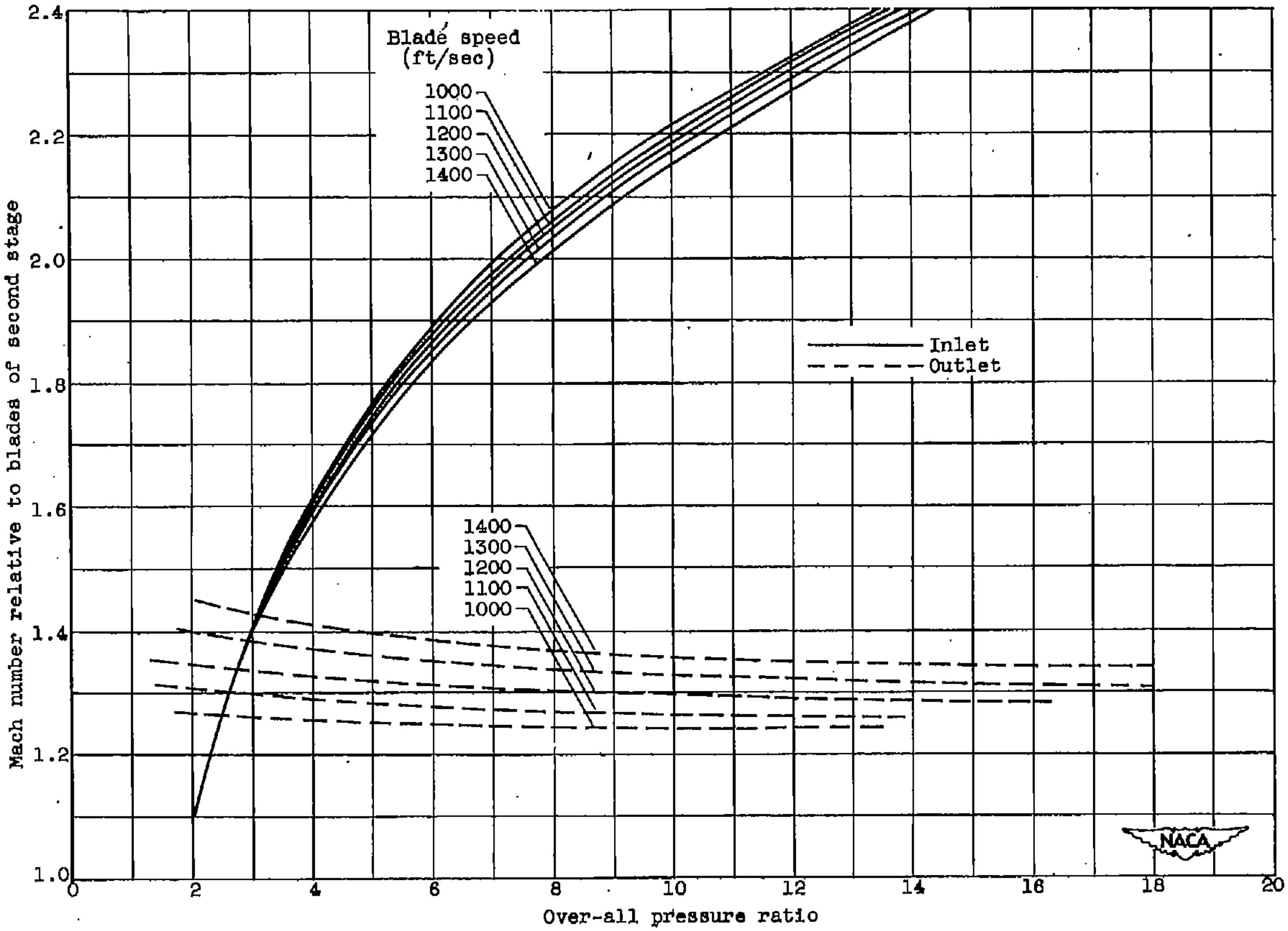

Figure 8. - Variation of inlet and outlet Mach numbers relative to blades of second stage with over-all pressure ratio. Case 2.

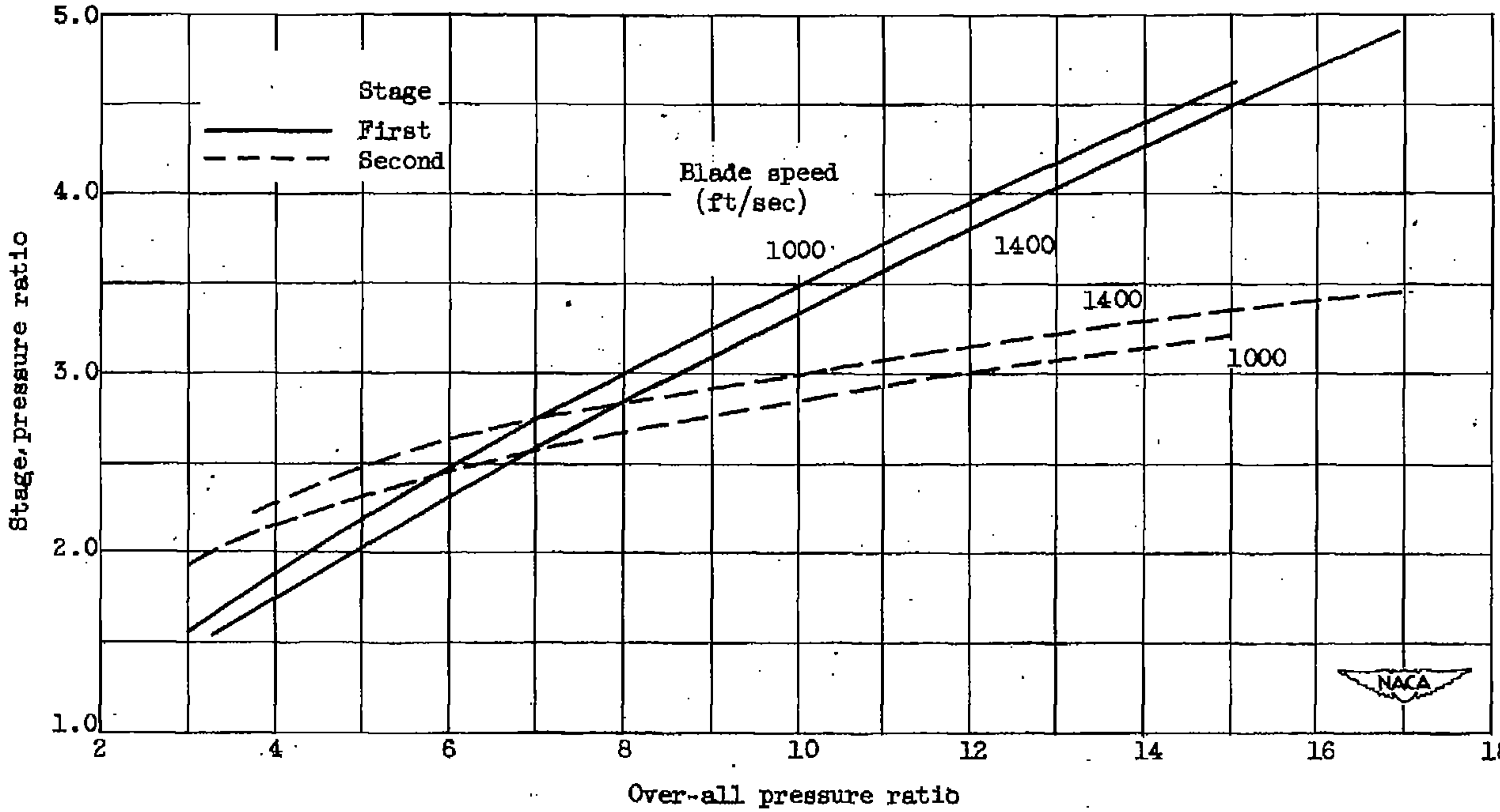

Figure 9. - Distribution of pressure ratio between stages. Case 2.

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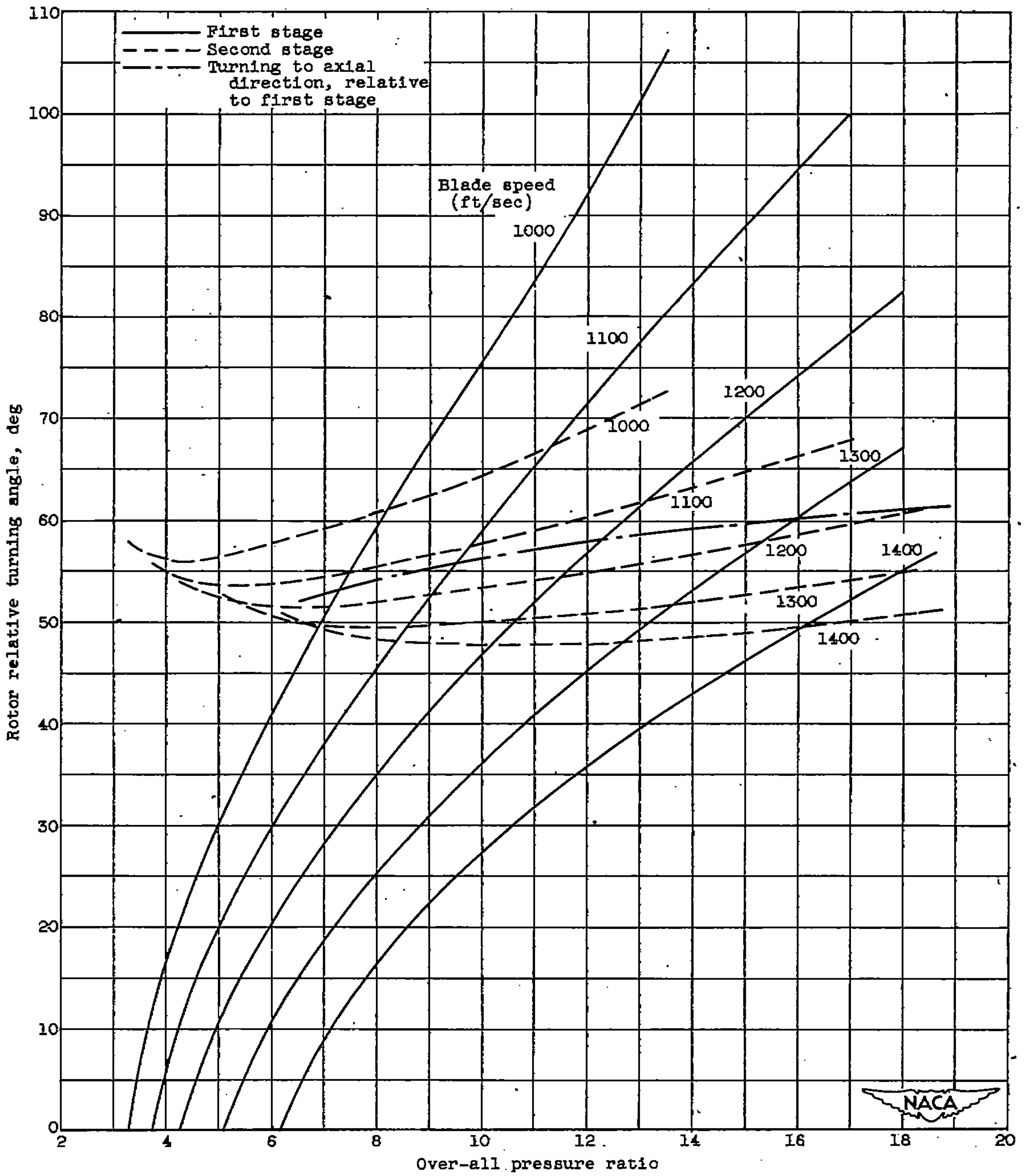

Figure 10. - Variation of rotor relative turning angle with over-all pressure ratio. Case 3.

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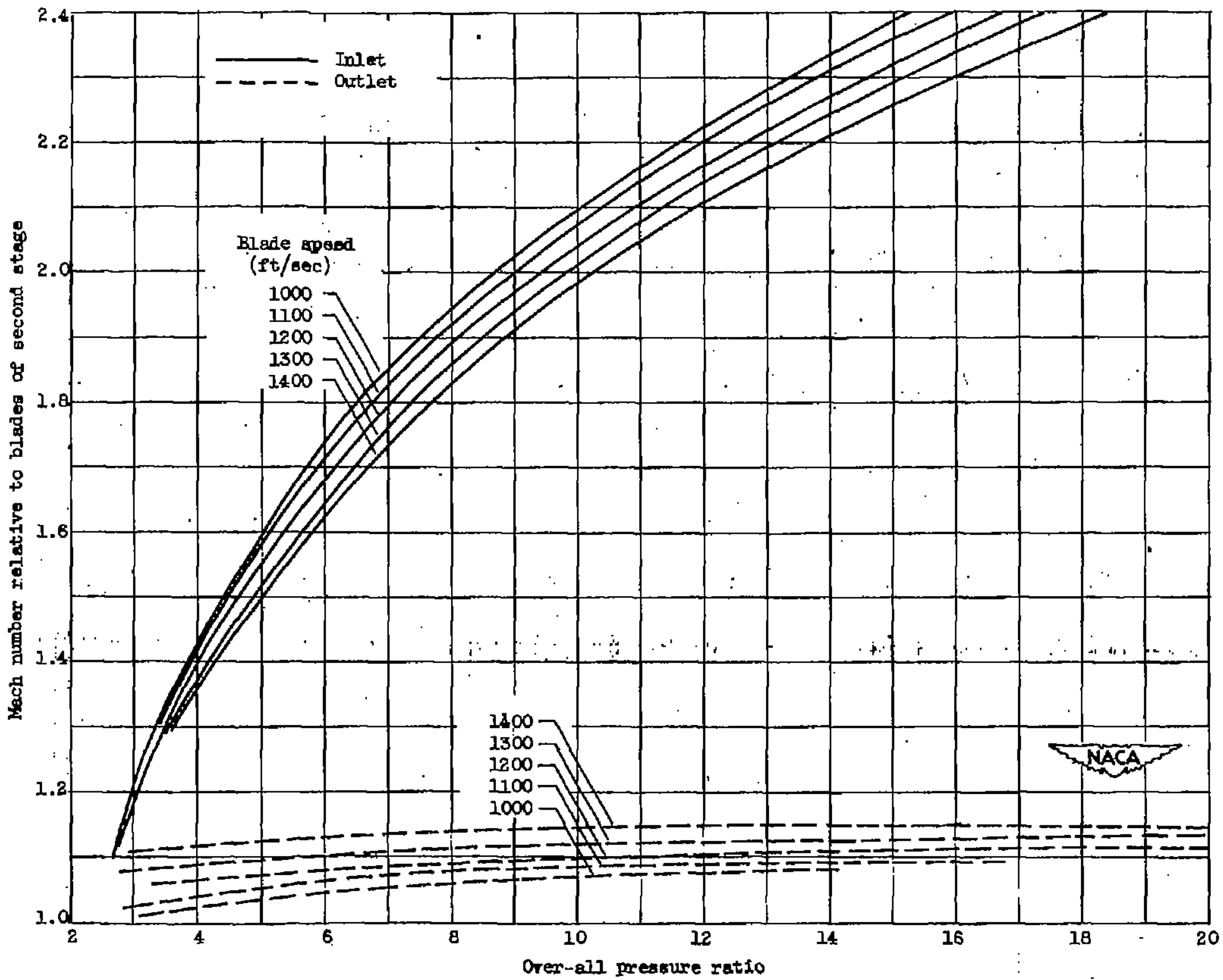


Figure 11. - Variation of inlet and outlet Mach numbers relative to blades of second stage with over-all pressure ratio. Case 3.

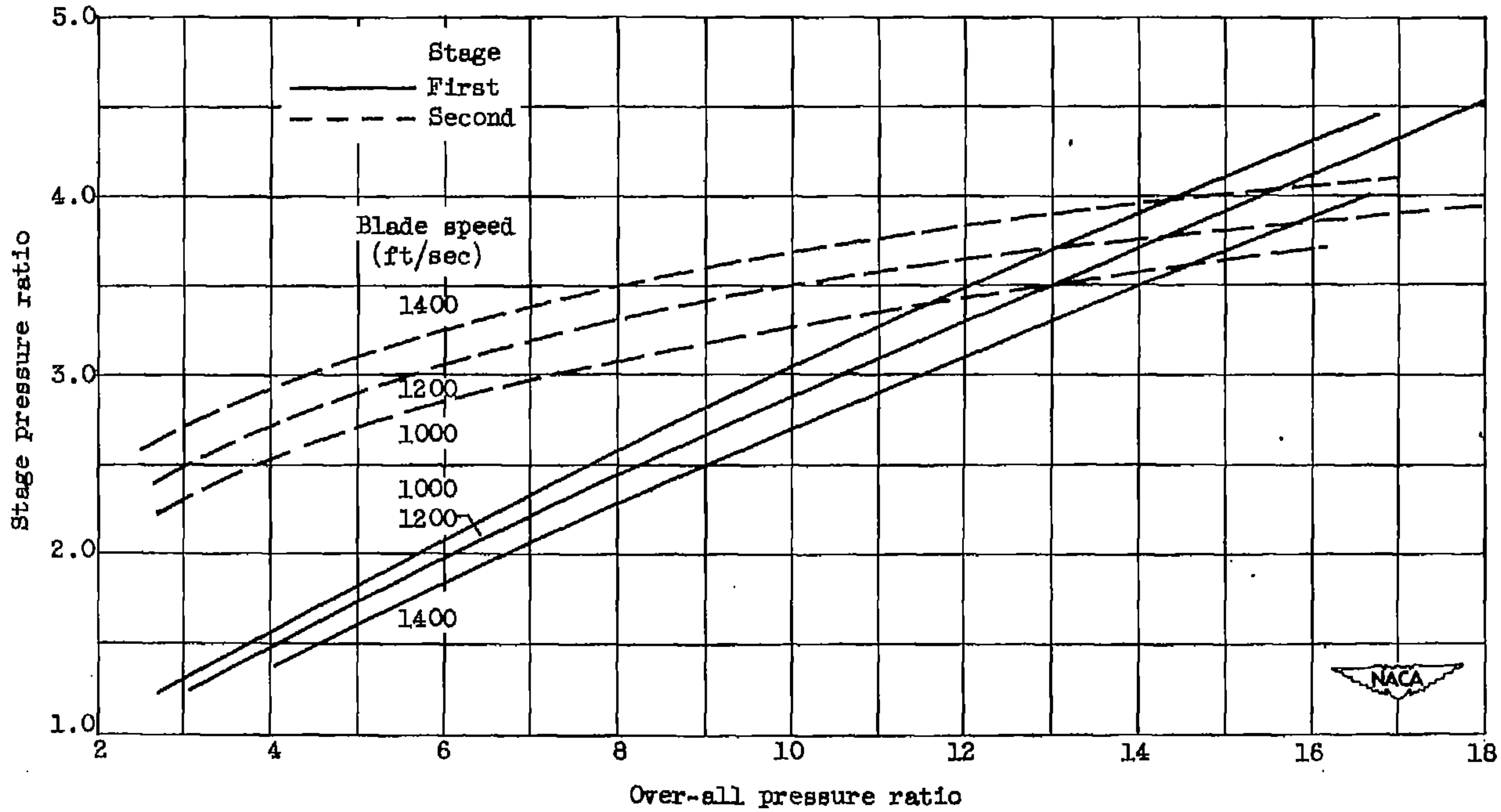
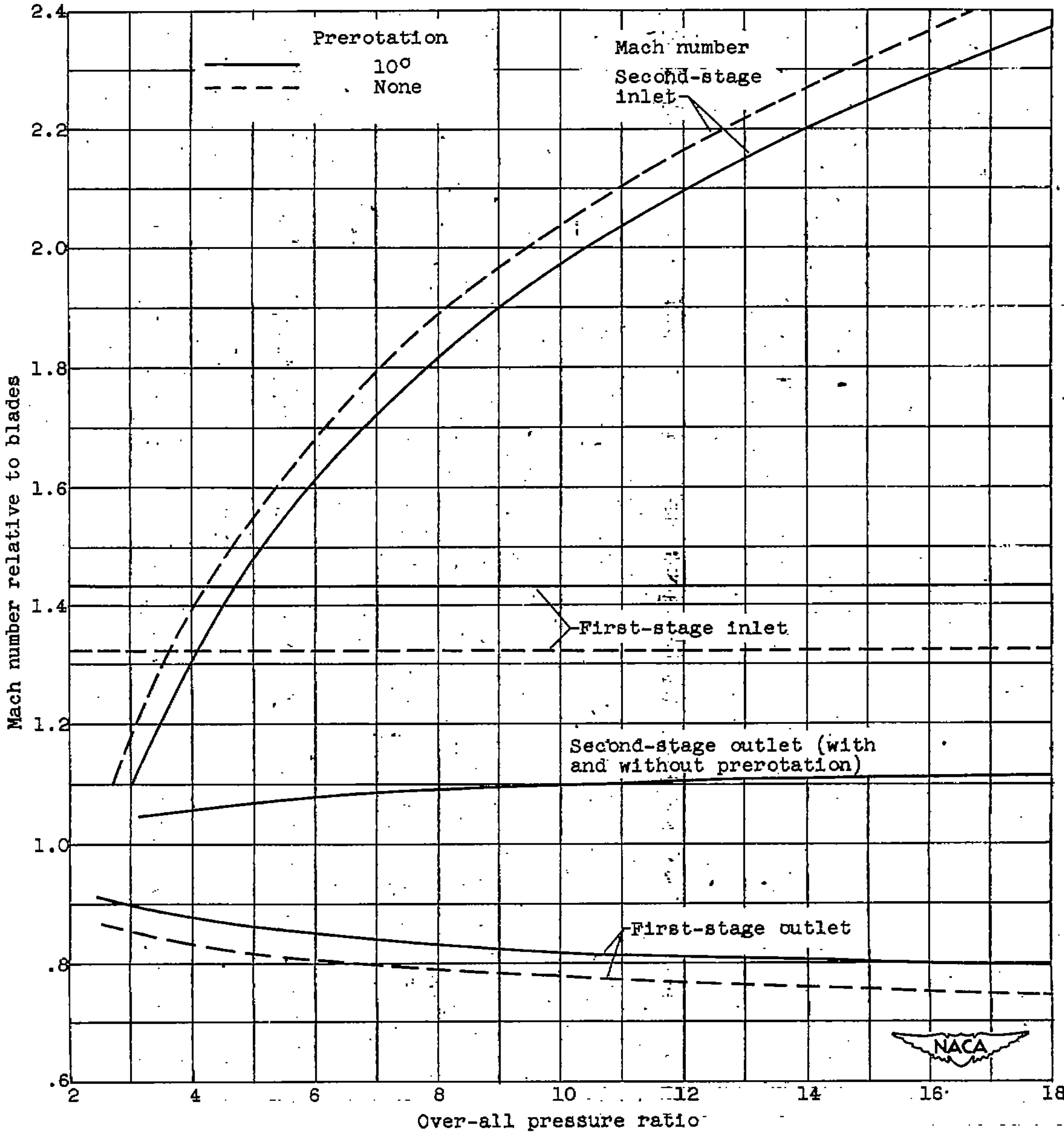


Figure 12. - Distribution of pressure ratio between stages. Case 3.



(a) Relative Mach number.

Figure 13. - Comparison of first and second stages with and without 10° prerotation. Case 3; blade speed, 1200 feet per second.

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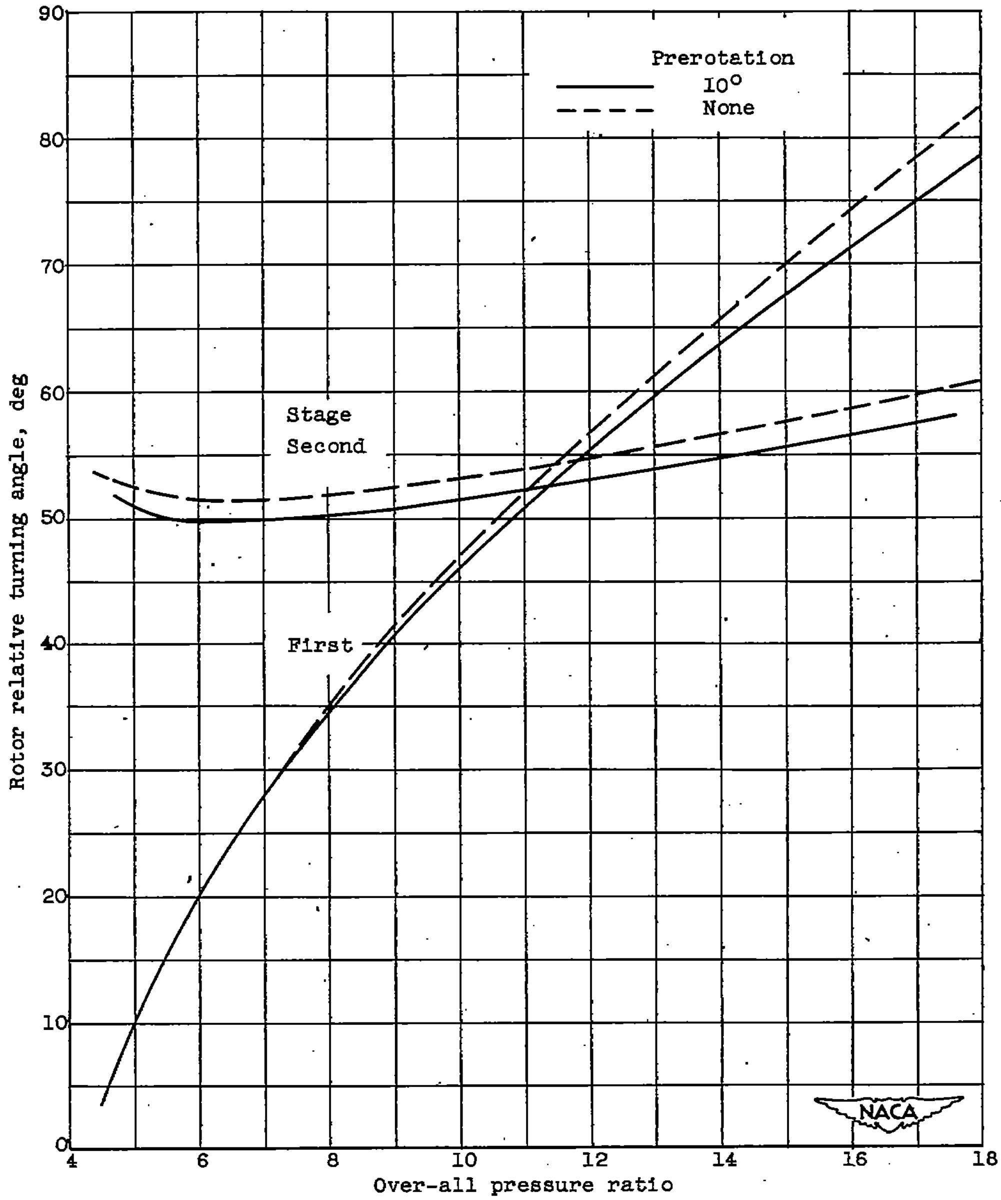
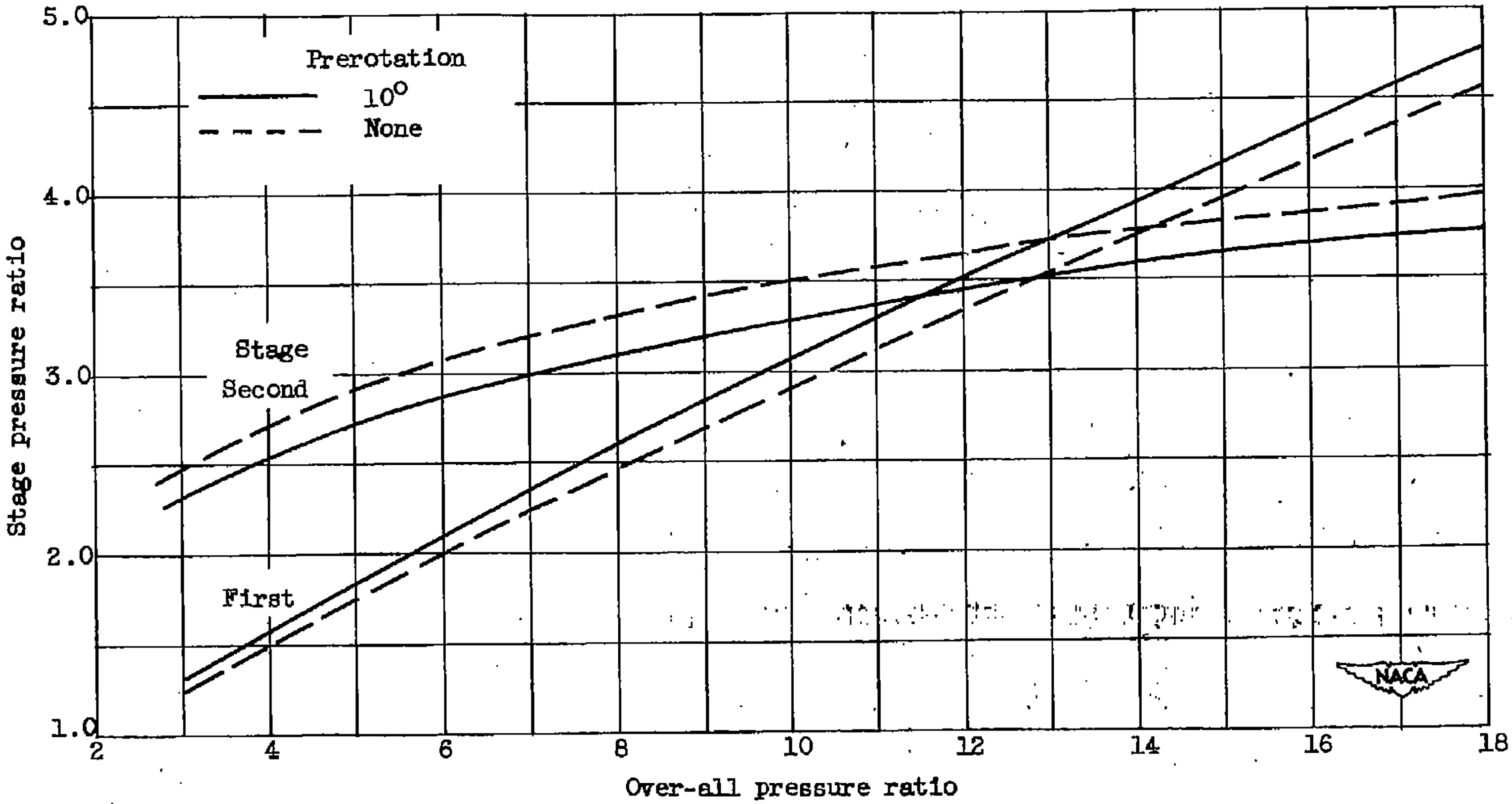


(b) Relative turning angle.

Figure 13. - Continued. Comparison of first and second stages with and without 10° prerotation. Case 3; blade speed, 1200 feet per second.



(c) Pressure ratio distribution.

Figure 13. - Concluded. Comparison of first and second stages with and without 10° prerotation. Case 3; blade speed, 1200 feet per second.

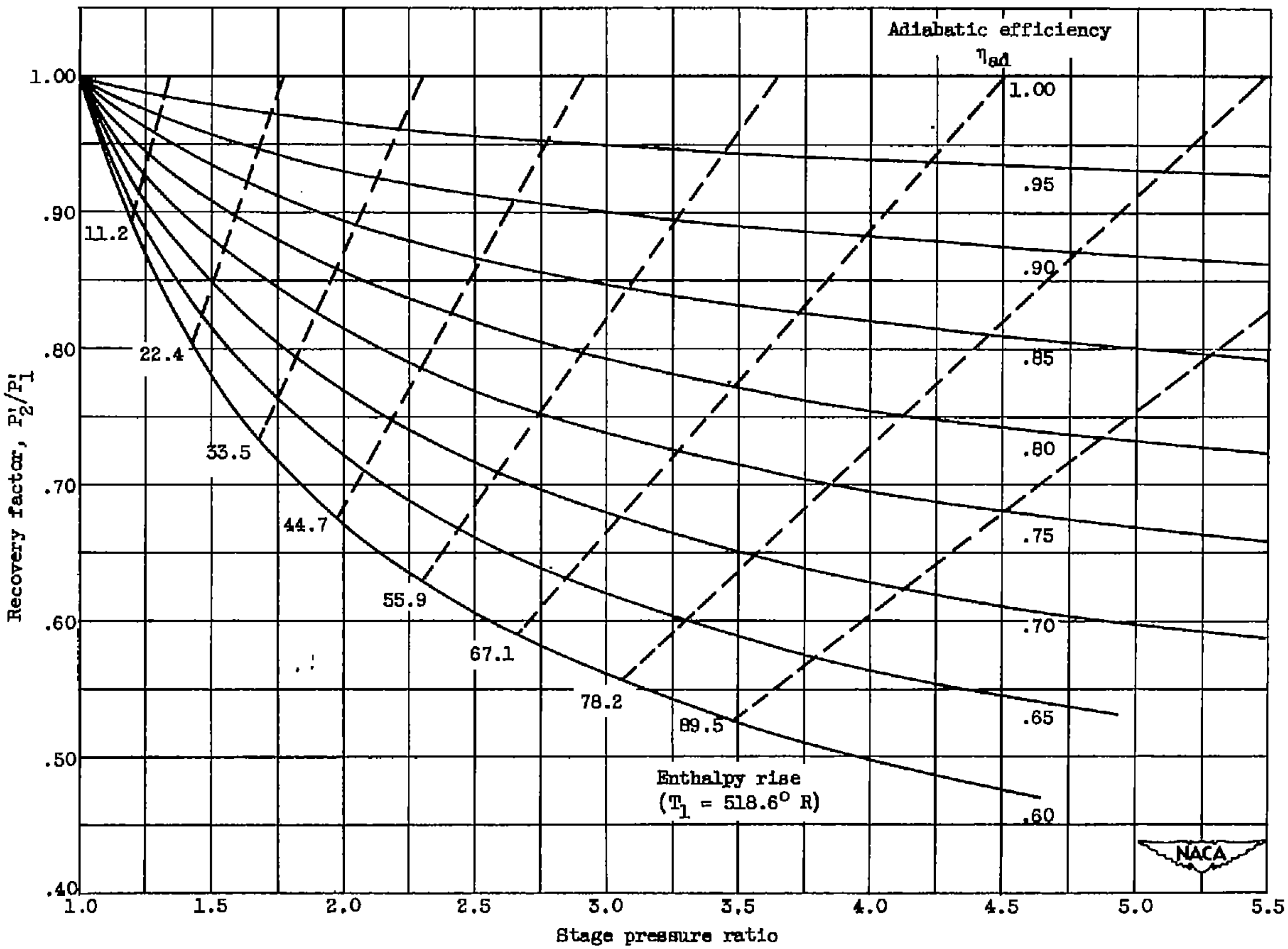


Figure 14. - Relation between recovery factor and adiabatic efficiency at various pressure ratios. One-dimensional flow.